COOLING SYSTEM TRANSIENT ANALYSIS USING AN ELECTRIC CIRCUIT PROGRAM ANALOG

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Abstract

By exploiting the common mathematical similarities between electrical and thermal circuits, an electric circuit transient analysis program may be used to analyze the transient behavior of thermal circuits. This is an especially convenient approach for the analysis and optimization of the cooling performance of burst mode type electrical apparatus such as high power transmitters. The relations between electrical components and the equivalent thermal components are established such that given the thermal component characteristics, one may then write and electric circuit with the proper values and then run the model to determine the performance in electrical parameters that are then easily interpreted as thermal parameters. Similar techniques have been used for the thermal analysis of components such as semiconductor devices [1] and analog hardware [2]. In this paper the P-Spice[TM] program is used as an example; however, the same procedures may be implemented using any electric circuit analysis program.

I. ELECTRO-THERMAL ANALOGS

A. *Objective*

In general, an electric circuit or a thermal circuit with n energy storage elements is described by an nth order differential equation. Electric circuit analysis programs provide the required solution by means of finite element analysis techniques. It is the objective here to establish an analog relation between a thermal circuit and an electric circuit such that the solution of the analog electric circuit provides a solution of the thermal circuit.

B. *Electro-Thermal Component Analogs*

In order to set up an electro-thermal analog, we need to identify the correspondence between the electrical and thermal parameters and variables. Because of the duality option in electric circuit analysis, we can use either inductance or capacitance to correspond to thermal mass. We choose capacitance as the analog of thermal mass. This choice imposes the interpretation of electric current as power flow and the voltage as temperature. This analog model pair is illustrated in Figure.1
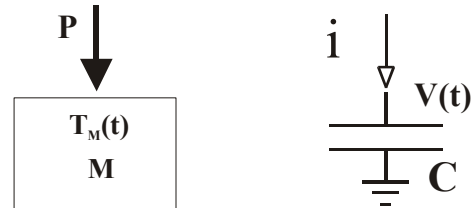

Figure 1. Thermal Mass, Capacitor Analog Model

The time dependent equations that describe the variables in Fig 1, are:

$$T_M(t) = T_M(0) + \int \frac{P}{M} dt \qquad (1)$$

$$V_C(t) = V_C(0) + \int \frac{i}{C} dt \qquad (2)$$

where:
T_M=temperature of thermal mass M, $^{\circ}$C
P =Power input to the thermal mass M, Watts
M=Thermal mass, Joules/, $^{\circ}$C
V_C=voltage on capacitance C, volts
i =current input to capacitance C, amps
C= value of capacitance, farads

Comparing the equations (1) and (2) we find that temperature corresponds to voltage, power (thermal) corresponds to current, and thermal mass corresponds to capacitance. Had we chosen to use inductance as the analog of thermal mass, then we would have found current as the analog of temperature and voltage that of power. Having chosen capacitance as thermal mass, then there is no use for inductance in the electrical analog of a thermal circuit.

Since current is the analog of power, then an electrical current source can be the analog of either a heat power source or a heat exchanger, depending upon which direction the current is introduced into the circuit.

Let us consider a liquid to air heat exchanger, commonly called a radiator, which is characterized by a Q that relates the power transferred in terms of the difference between the inlet air and coolant temperature. In general the Q

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14. ABSTRACT
By exploiting the common mathematical similarities between electrical and thermal circuits, an electric circuit transient analysis program may be used to analyze the transient behavior of thermal circuits. This is an especially convenient approach for the analysis and optimization of the cooling performance of burst mode type electrical apparatus such as high power transmitters. The relations between electrical components and the equivalent thermal components are established such that given the thermal component characteristics, one may then write and electric circuit with the proper values and then run the model to determine the performance in electrical parameters that are then easily interpreted as thermal parameters. Similar techniques have been used for the thermal analysis of components such as semiconductor devices [1] and analog hardware [2]. In this paper the P-SpiceTM program is used as an example; however, the same procedures may be implemented using any electric circuit analysis program.

15. SUBJECT TERMS

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depends upon the particular unit and the air and coolant flows. The characteristics of a typical commercial heat exchanger are shown in Figure 3.
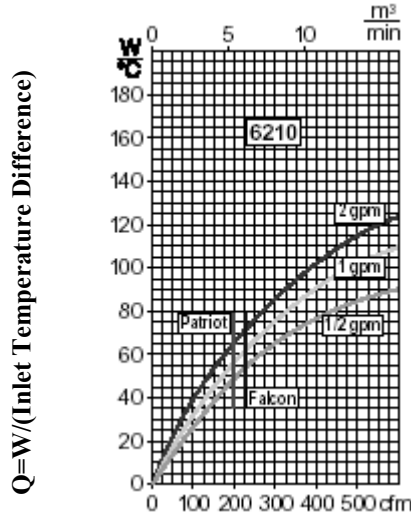

Figure 3. Characteristics of a typical liquid to air radiator.

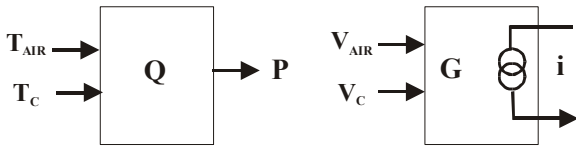The electric analog is implemented by a voltage controlled current source as shown in Figure 4.

Figure 4. Radiator-Current Source Analog Model

The equations relating to Figure 4 are:

$$P = Q(T_C - T_{AIR}) \qquad (3)$$
$$i = G(V_C - V_{AIR}) \qquad (4)$$

The equations are arranged such that, if the air temperature is greater than the coolant, then the power is transferred to the air and has a positive sign.

A resistor in an electrical circuit can be interpreted as the analog of heat being transferred by coolant flow in a thermal circuit. A diagram of this component pair is shown in Figure 5.
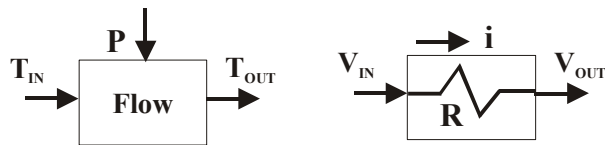

Figure 5. Coolant Flow-Resistor Analog for Heat Transport

In order to relate the value of R to the "Flow" of the coolant, we must consider the specific heat as well as the mass or volumetric flow of the coolant. The power transported by the flow is given by:

$$P = f\delta\sigma(T_{OUT} - T_{IN}) \qquad (5)$$

where:
  f=volumetric flow of coolant, liters/second
  δ=density of coolant, kg/liter
  σ=specific heat of coolant, Joules/kg-degree C

The current flow in the electric circuit is:

$$i = \frac{(V_{IN} - V_{OUT})}{R} \qquad (6)$$

Comparing (5) and (6), the value of R is determined as:

$$R = \frac{1}{f\delta\sigma} \qquad (7)$$

The flow of coolant in the electric circuit is imbedded in the value of the resistor. Confusion can arise if one attempts to associate coolant flow in the configuration of the electric analog model of a cooling system, in that the flow of the coolant appears to be opposite from the flow of heat power. This confusion is a result of determining if the power is being removed or added, and from what or to what.

We have now established enough thermal-electric analog components to model a complex cooling system consisting of multiple thermal masses, heat sources, heat exchangers and other elements.

II. APPLICTION EXAMPLES

We shall first implement a simple example of the cooling dynamics of a high power electron beam device and then expand to a more complicated example of an entire transmitter.

A. Thermal Model of Electron Beam Power Device
Liquid cooled high power electron beam devices such as klystrons, gyrotrons, etc, have massive collectors that account for nearly all of the power dissipation. Under pulse or burst mode operation the thermal mass of the collector as well as the coolant flow is an important consideration in the analysis of the thermal performance. A simple electrical analog model of a pulsed beam device is shown in Figure 6. A pulsed voltage source, V(t), gates a current source with transconductance G that simulates the power dissipation of the collector. The mass of the collector is modeled by the capacitor C and the coolant analog is imbedded in the resistor R.

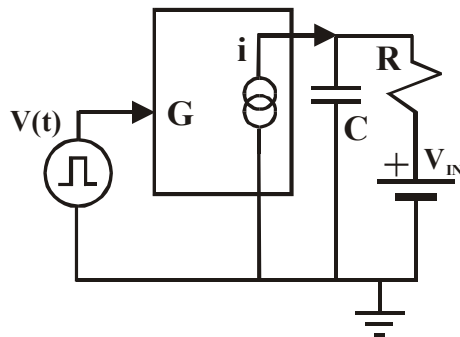The dc voltage source, V_{IN}, models the inlet temperature of the coolant.

Figure 6. Electrical Analog of Collector of a Pulsed Beam Power Device

To better understand the example, the following numeric values are assigned to the parameters. The power input to the collector is a 50kW pulse with a one second duration and a three second period. The collector mass is 50kg of copper with a specific heat of 380J/kg°C thus the value of C in the electric circuit is 19000Farads. The collector coolant is water with a flow of 0.125liters/second and an inlet temperature of 60°C. The value of the R from equation (7) is 1.9138mΩ. Initially the temperature of the collector is stabilized at the coolant inlet temperature. The circuit in Figure 6 as written in P-Spice is shown in Figure 7.
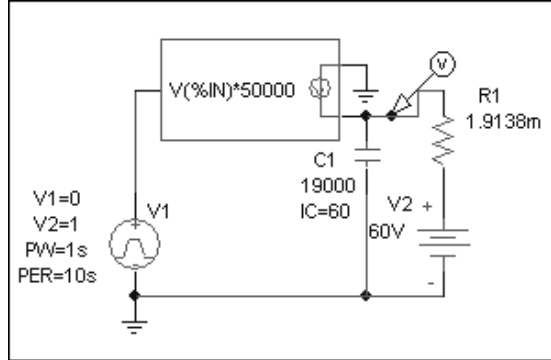

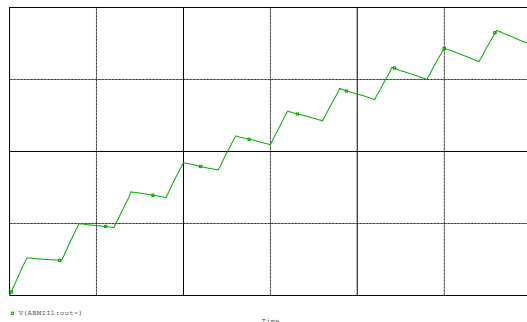**Figure 7.** P-Spice Circuit of Figure 6 Model

Figure 8. Collector Temperature as Determined by the Model in Figure 7, Scales; Time=0 to 30 Seconds, Temperature 60C to 80C (Volts)

B. Example of a Transmitter With Cooling System

The P-Spice model of a beam power device transmitter including the cooling system is shown in Figure 10. The waveform begins with a 30 second pulse followed by a 30 pulse burst of 10 Second pulses with a 20 Second period, then followed by a 60 second off period. The peak collector dissipation is 50kW with the same mass as the previous example. The cooling system has a water capacity of 10kg, a flow of 0.0478liters/second and a radiator with a Q=2400W/°C. The ambient air temperature and the initial temperatures of the water and the collector are 40 °C.

The waveform is generated by two pulse generators and a logic statement in the dual voltage controlled current source. The first pulse generator produces a single initial pulse of 30 seconds. The second generator produces a repped 10 second pulse with a period of 20 seconds after an initial 50 second delay. The logic statement in the current source blanks any output after 150 seconds.

The inlet water temperature of the radiator, modeled by a dual voltage input current source, is the collector temperature or equivalently the water exit temperature from the collector. The air temperature input to the radiator is the ambient air temperature, Tair=60.

The radiator removes heat from the coolant and dumps it into the ambient air at 60C. In the model this is represented by a current being removed from the capacitor C2 and then sunk to ground as shown in Figure 10.

The resistor, R1, models the collector coolant flow and the transfer of heat from the collector to the coolant reservoir.

The thermal mass of the coolant, 10kg=41800Joules/ °C is modeled by C2 set to 41800 Farads.

The results of the simulation are shown in Figure 9.

III.SUMMARY

We have shown that an electric circuit simulation program can be used to analyze the transient thermal response of heat flow circuits. The complexity of the thermal circuit may be much greater that the examples given. By following the analog simulation rules the only limit to the number of heat sources, thermal masses, flow loops, heat exchangers, and other components is that of the simulation program which typically extends to hundreds of components.
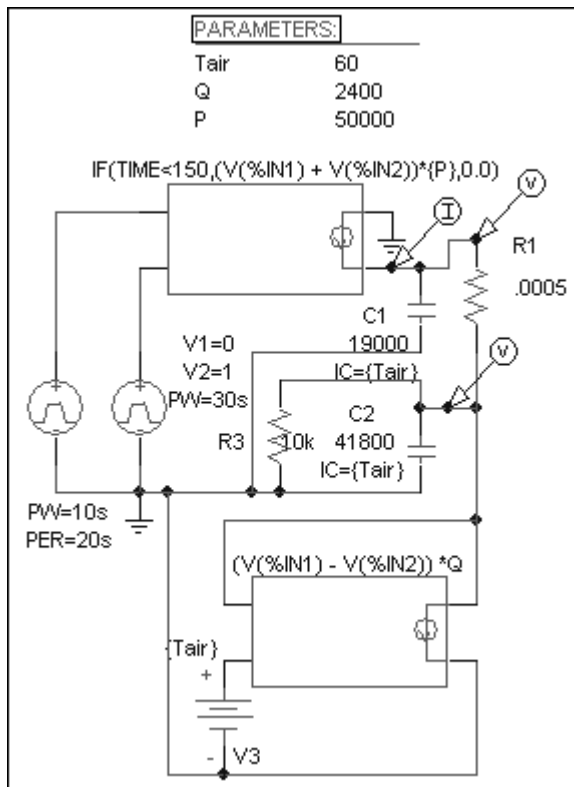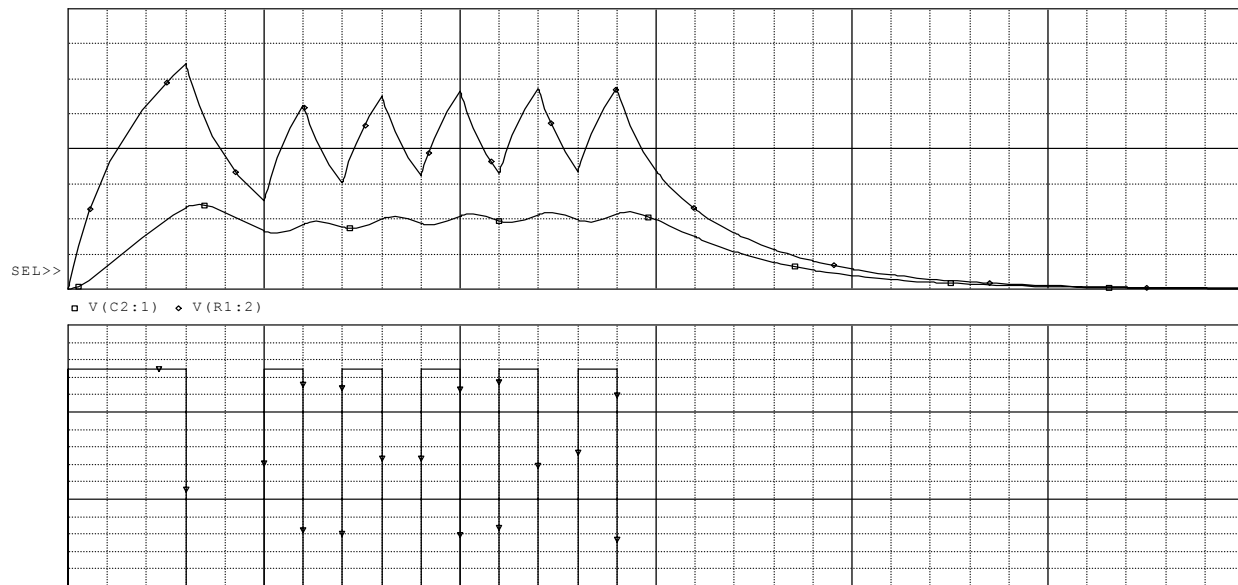
Figure 10. P-Spice Model of Transmitter Including the Cooling

IV.REFERENCES

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[2] X. Huang and H. A. Mantooth, "Event-Driven Electrothermal Modeling of Mixed-Signal Circuits", in Proc. 37th Design Automation Conference, pp.10-15, June 2000.